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Hadron masses in qQCD with Wilson fermions near the chiral limit[†]

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Abstract

In quenched lattice QCD with standard Wilson fermions the quark propagator is computed very close to the chiral limit in the zero-temperature case. Starting from our experience with lattice QED we employ a modified statistical method in order to estimate reliably hadron masses.

1 Introduction

Chiral symmetry has remained a problematic topic in lattice gauge theories over the years. As is well-known, Wilson fermions explicitly violate the chiral symmetry, which can be hopefully restored in the limit $\kappa \rightarrow \kappa_c(\beta)$ (β and κ denote the gauge coupling and hopping parameter, respectively). The QCD study at the experimentally established m_π/m_ρ value is obstructed

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by a huge amount of required computer resources and – particularly for the quenched approximation – by large fluctuations in the observables caused by near-to-zero eigenvalues of the fermion matrix (‘exceptional configurations’).

In [1] a variance reduction technique for estimating the pseudoscalar mass m_π was proposed. In the case of QED the new estimator for m_π was proved to work well close to κ_c , where the standard estimator fails. In this work we extend the study to the case of quenched lattice QCD.

The Wilson action for lattice QCD is $S = S_G + S_F$, where S_G denotes the plaquette $SU(3)$ gauge action and S_F the fermion part

$$S_F(U, \bar{\psi}, \psi) = \sum_{f=1,2} \sum_{x,y} \bar{\psi}_x^f \mathcal{M}_{xy}(U) \psi_y^f, \quad (1)$$

with the Wilson matrix $\mathcal{M}(U) = \hat{1} - \kappa \mathcal{D}(U)$,

$$\mathcal{D}_{xy} \equiv \sum_{\mu} \left[\delta_{y,x+\hat{\mu}} P_{\mu}^{-} U_{x\mu} + \delta_{y,x-\hat{\mu}} P_{\mu}^{+} U_{x-\hat{\mu},\mu}^{\dagger} \right] \quad (2)$$

and $P_{\mu}^{\pm} = \hat{1} \pm \gamma_{\mu}$.

The observables of interest are the π and ρ -meson correlators (masses). On every configuration $\{U\}$ we determine the operators

$$\Gamma_{[\pi,\rho]}(\tau) \equiv \frac{1}{N_s^6} \sum_{\vec{x},\vec{y}} \text{Tr} \left(\mathcal{M}_{xy}^{-1} \gamma_{[5,\mu]} \mathcal{M}_{yx}^{-1} \gamma_{[5,\mu]} \right) \quad (3)$$

where $\tau = |x_4 - y_4|$.

All simulations were performed on a $16^3 \times 32$ lattice at $\beta = 6.0$. The number of the gauge field configurations is $O(100)$. κ -values were chosen between 0.1558 and 0.1570. The inversion of \mathcal{M} was done with BiCGstabl with restarting.

2 Correlators and Masses

The standard definition of the effective mass $m_{eff}(\tau)$ is given by the equation

$$\frac{\cosh(m_{eff}(\tau)(\tau + 1 - \frac{L_4}{2}))}{\cosh(m_{eff}(\tau)(\tau - \frac{L_4}{2}))} = \frac{\langle \Gamma_{[\pi,\rho]}(\tau + 1) \rangle}{\langle \Gamma_{[\pi,\rho]}(\tau) \rangle}. \quad (4)$$

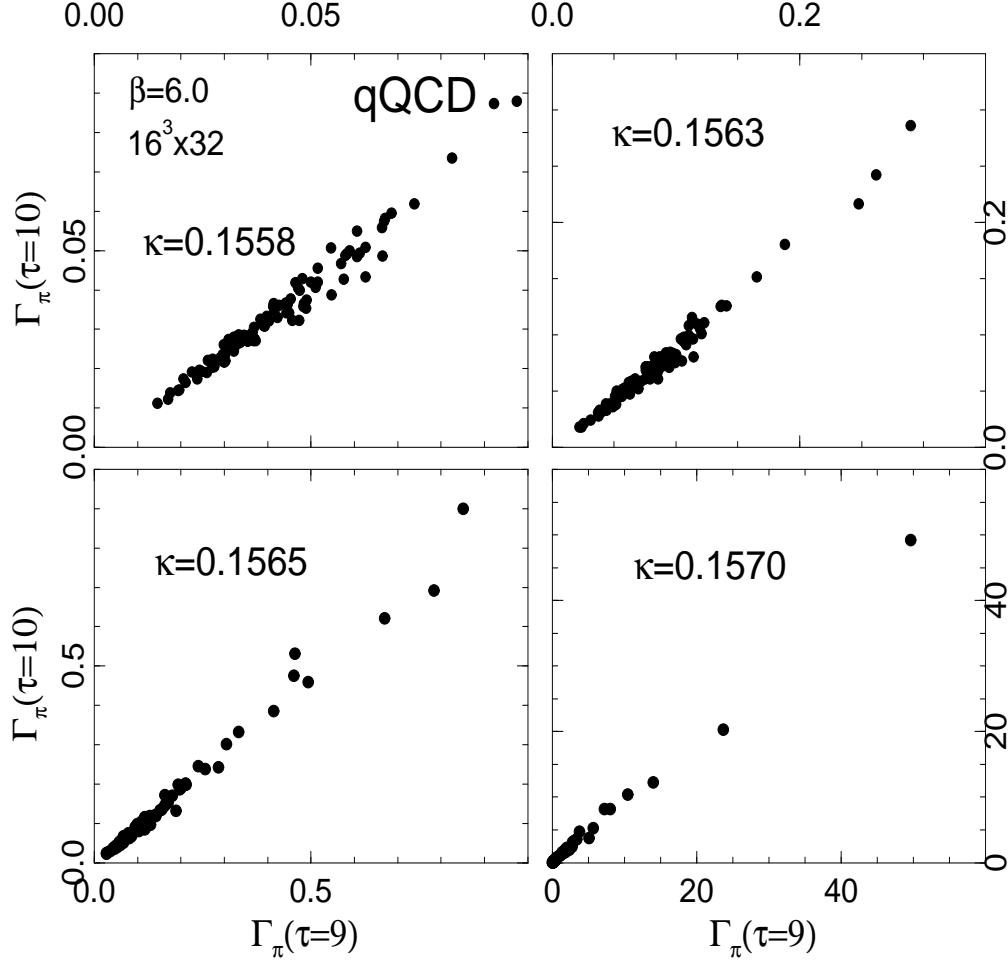


Figure 1: Scatter plots for the pion correlator.

The plateau in the τ -dependence of $m_{eff}(\tau)$ is supposed to define the ‘true’ mass $m_{[\pi,\rho]}$. However, for $\kappa \rightarrow \kappa_c(\beta)$ the values of $\Gamma_\pi(\tau)$ become strongly fluctuating. There is no reliable estimate for the averages $\langle \Gamma_\pi(\tau) \rangle$, the approach to $\kappa_c(\beta)$ seems extremely difficult [2, 3].

In [1] it was shown that a new estimator for the pseudoscalar mass can be defined which is identical to the standard estimator in the case of linear correlations

$$\overline{y}(x) = C \cdot x, \quad (5)$$

where $x \equiv \Gamma_\pi(\tau)$; $y \equiv \Gamma_\pi(\tau + k)$; $k > 0$, and $\Gamma_\pi(\tau + k)$, $\Gamma_\pi(\tau)$ are calculated on individual configurations. $\overline{y}(x)$ stands for the conditional average of $y(x)$

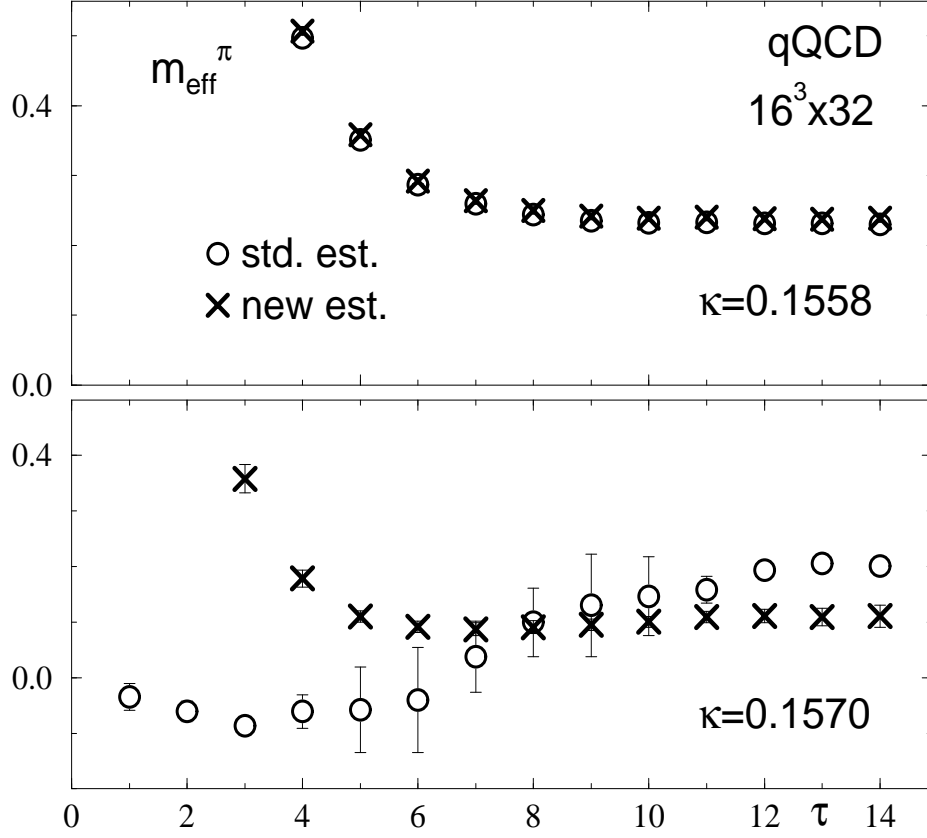


Figure 2: m_{eff}^π over τ for two values of κ . (o) denote the standard and (x) the improved estimator.

at a fixed value of x . If (5) holds with $x \neq 0$, then

$$\frac{\langle \Gamma_\pi(\tau + k) \rangle}{\langle \Gamma_\pi(\tau) \rangle} \equiv \frac{\langle y \rangle}{\langle x \rangle} = \left\langle \frac{y}{x} \right\rangle \equiv \left\langle \frac{\Gamma_\pi(\tau + k)}{\Gamma_\pi(\tau)} \right\rangle, \quad (6)$$

and the effective masses are determined by the r.h.s. of (6). The important observation [1] is that the ratio $\Gamma_\pi(\tau + k)/\Gamma_\pi(\tau)$ calculated on individual configurations does not suffer from near-to-zero eigenvalues. The average of this ratio is statistically well-behaved, in contrast to the ratio of averages.

A typical situation is shown in Fig.1 where $\Gamma_\pi(\tau+1)$ over $\Gamma_\pi(\tau)$ is depicted for $\tau = 9$ at several κ 's (note the different scales). We do not find any qualitative difference to the quenched compact QED₄ case. Therefore, we expect the improved estimator to be applicable also for quenched QCD.

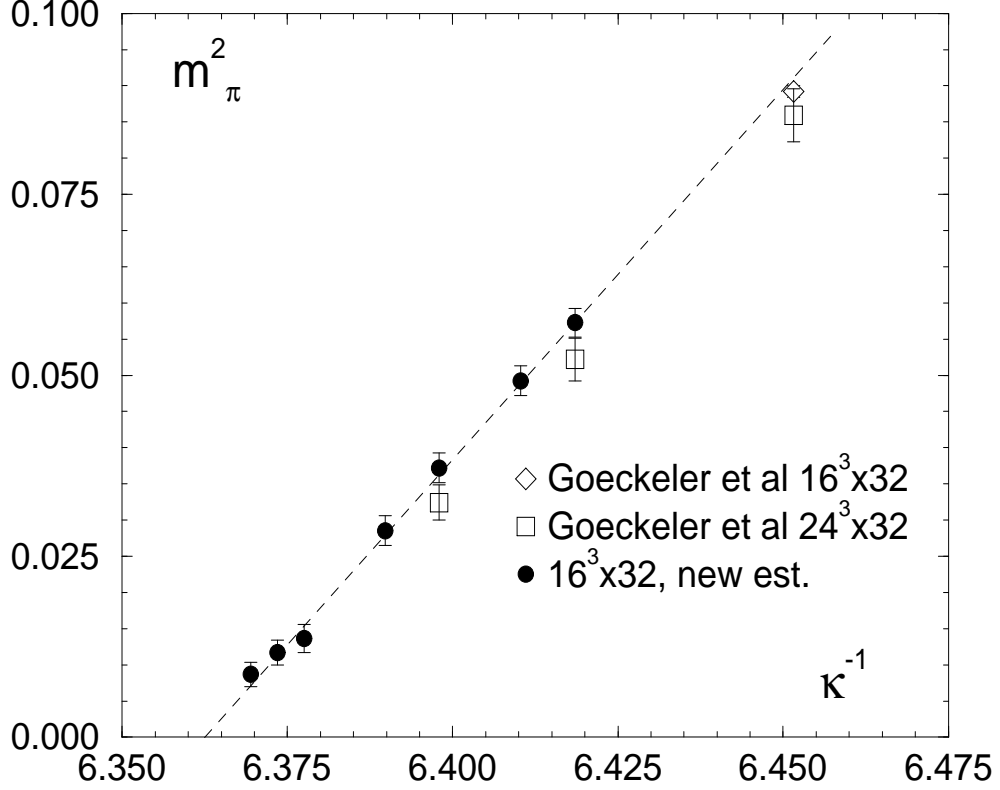


Figure 3: m_π^2 as a function of $1/\kappa$.

Fig.2 represents the effective mass $m_{eff}^\pi(\tau)$ for two different values of κ . For the smaller κ we obtain very good agreement between the two estimators, which with more statistics, should become identical. In the lower plot, near to κ_c , the standard estimator and its error bars cannot be trusted anymore. In contrast, the improved estimator gives a very nice plateau in m_{eff}^π at $\tau > 5$ which still is statistically well-defined.

The extrapolation $m_\pi \rightarrow 0$ is shown in Fig.3. The PCAC-like dependence

$$m_\pi^2 \sim m_q = \frac{1}{2} \left(\frac{1}{\kappa} - \frac{1}{\kappa_c} \right) \quad (7)$$

is nicely fulfilled. In this figure we included also data from the QCDSF collaboration on $16^3 \times 32$ and $24^3 \times 32$ lattices [4] for comparison. The data look well-consistent for coinciding lattice sizes.

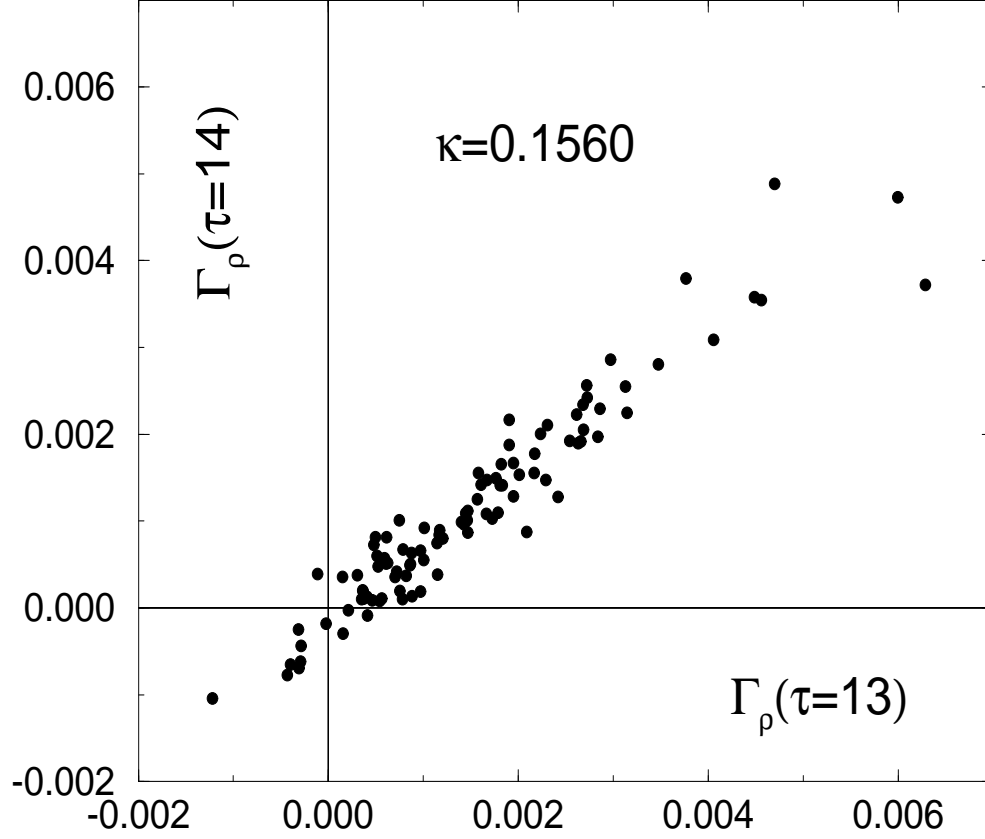


Figure 4: Scatter plot for ρ -meson correlator.

The ρ -meson mass m_ρ should not disappear in the limit $\kappa \rightarrow \kappa_c$. Therefore, the ρ -meson correlators $\Gamma_\rho(\tau)$ are expected to be less sensitive with respect to near-to-zero eigenvalues. The standard estimator for m_ρ works better, in contrast to the pion case. However, it is interesting to note that scatter plots for the ρ -meson correlators with different τ show again the linear correlations between different τ -slices (see Fig. 4).

3 Summary

We observed that due to strong linear correlations between the values of the pseudoscalar correlators at different τ the ratios $\Gamma_\pi(\tau+1)/\Gamma_\pi(\tau)$ do not suffer from near-to-zero (exceptional) eigenmodes of the fermionic matrix. This means that for every given configuration all ‘divergent’ contributions to the correlators are factorized.

Making use of this observation we proposed another estimator of the pseudoscalar mass, which is well-defined near $\kappa_c(\beta)$ in contrast to the standard estimator.

For κ -values sufficiently below $\kappa_c(\beta)$, i.e., where the standard estimator can be reliably defined, both estimators are shown to be in a very good agreement. By approaching $\kappa_c(\beta)$ we observe a PCAC-like dependence as in eq. (7). For values of κ very close to $\kappa_c(\beta)$ the standard estimator fails to work, while the improved one still fits the same straight line having a very small statistical error.

The new estimator permits to approach the chiral limit much closer than the standard one would allow. Especially, systematic errors induced by the finite volume and/or by the quenched approximation can be investigated in more detail within the ‘critical’ region. In particular, the ‘critical’ value $\kappa_c(\beta)$ can be determined with higher accuracy.

An alternative way to cure the problem of ‘exceptional configurations’ was proposed in Ref. [5].

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